3D Mapping Aided GNSS Using Gauss-Newton Algorithm: An example on GNSS shadow matching

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Session B4b: Applications of GNSS Measurements from Smartphones,
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Urban GNSS Positioning

Line-of-sight (LOS) signal:

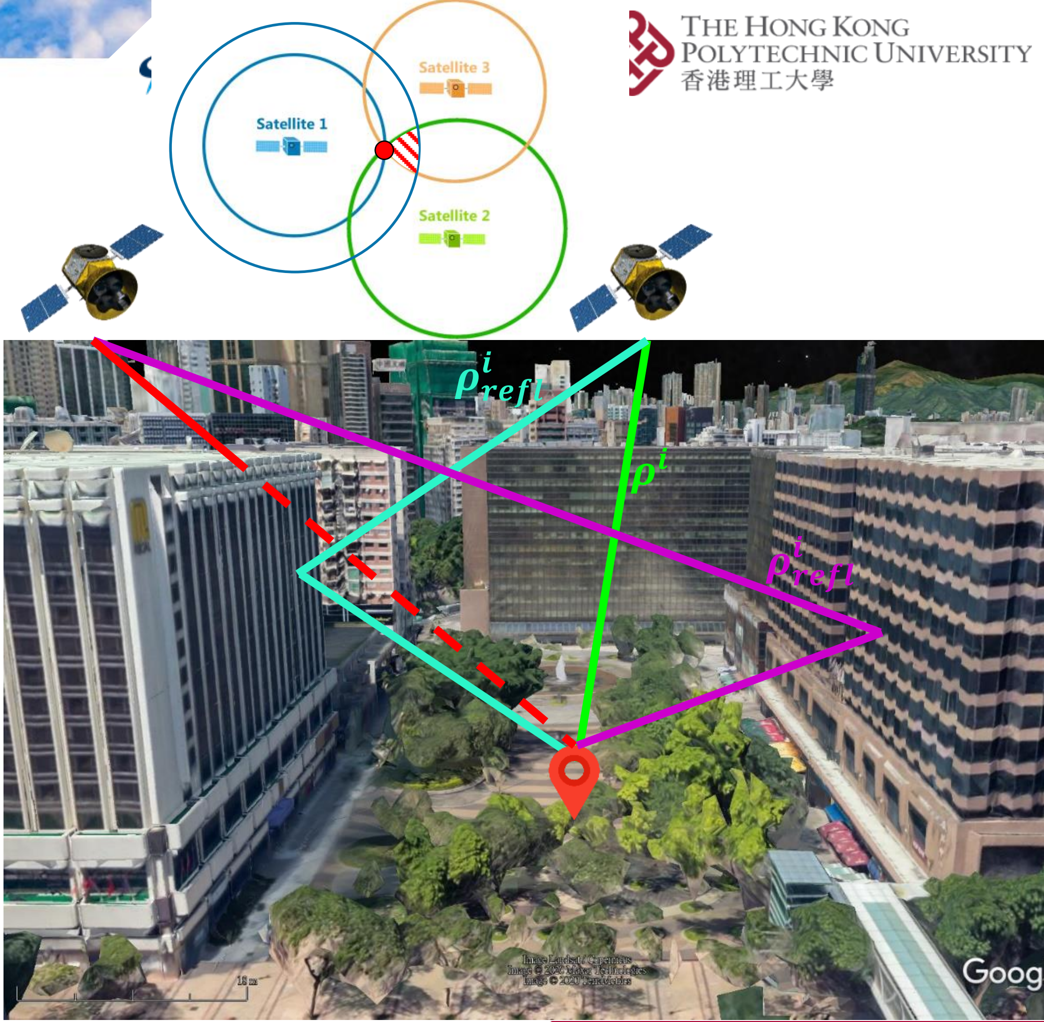
$$\rho^i = D^i + c\left(\delta t_r - \delta t^i\right) + T^i + I^i + \varepsilon^i$$

$$\phi^i = D^i + c\left(\delta t_r - \delta t^i\right) + T^i + I^i + \varepsilon^i + \lambda^i N^i$$

Reflected signal:

$$\rho^i_{refl} = D^i + c\left(\delta t_r - \delta t^i\right) + T^i + I^i + \varepsilon^i_{refl} + \varepsilon^i$$

$$\phi^i_{refl} = D^i + c\left(\delta t_r - \delta t^i\right) + T^i + I^i + \lambda^i N^i + \varepsilon^i_{refl} + \varepsilon^i$$

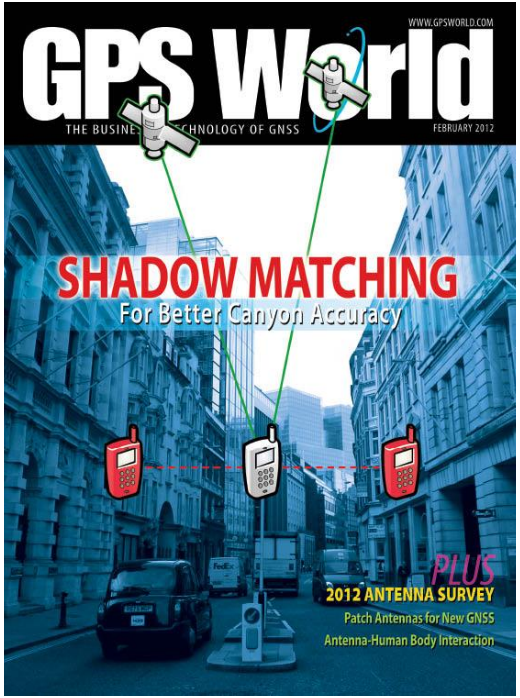


NLOS reception: LOS signal is blocked only receiving reflected signal

Multipath: receiving both LOS signal and reflected signal

Popular 3D Mapping Aided (3DMA) GNSS

Shadow matching (Satellite Visibility)



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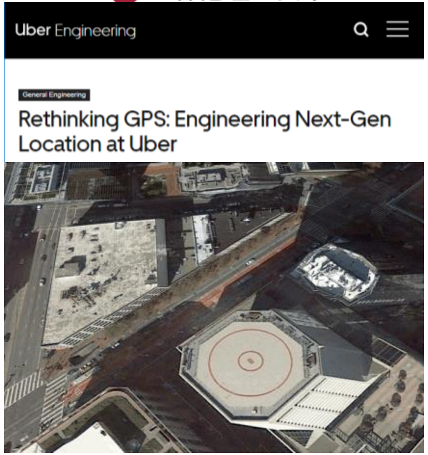
GNSS Ray-tracing (Range and C/N_0)

=

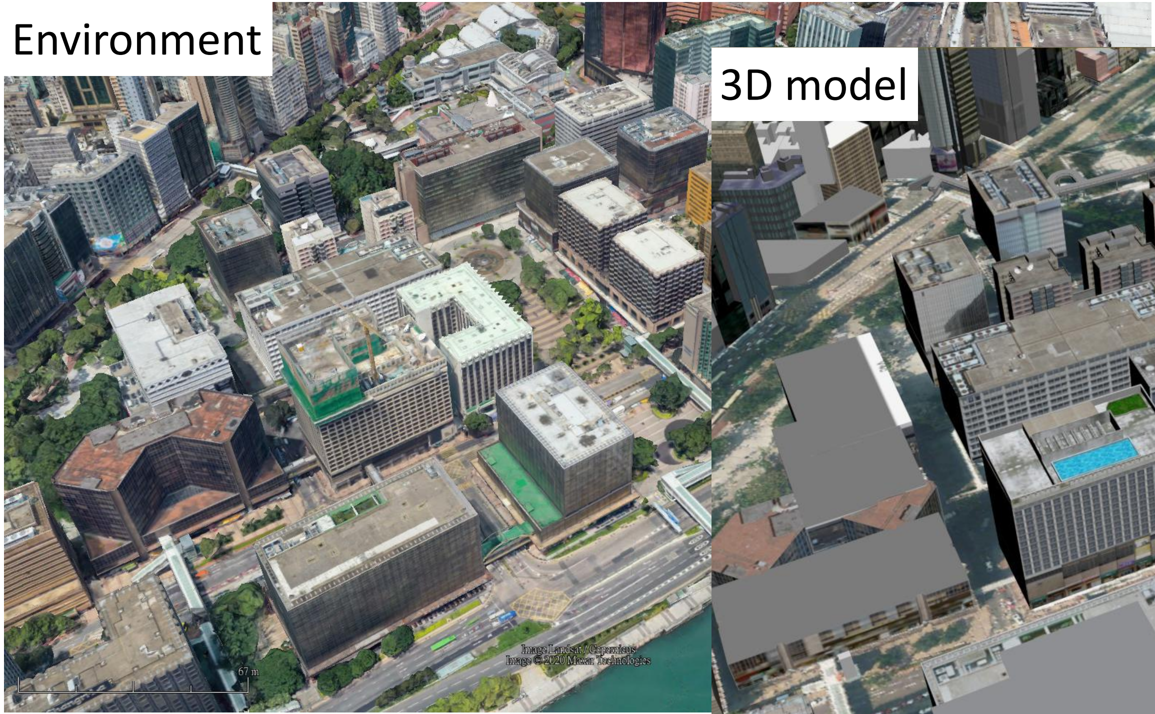


[Uber: Rethinking GPS: Engineering Next-Gen Location at Uber]

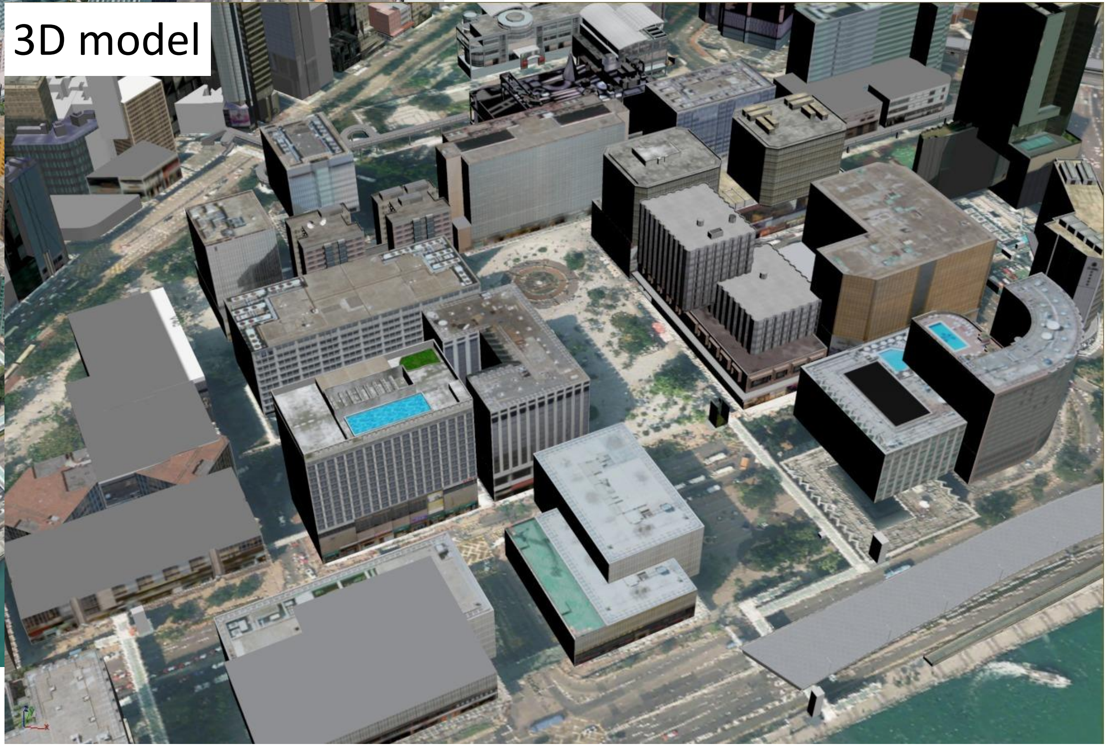
[Google: Improving urban GPS accuracy for your app]

Powerful Resources: 3D Building Model!

Environment

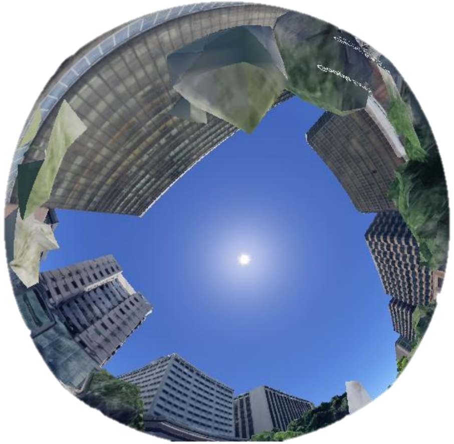


3D model

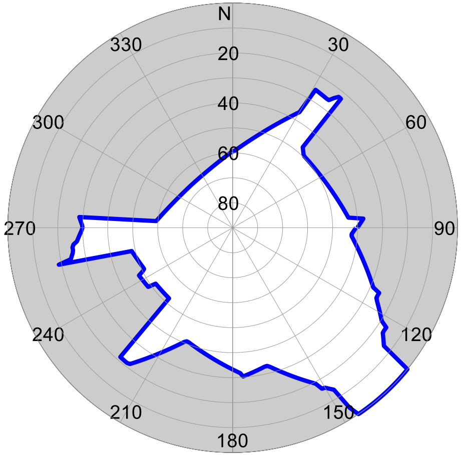


Powerful Resources: 3D Building Model!

Select a location

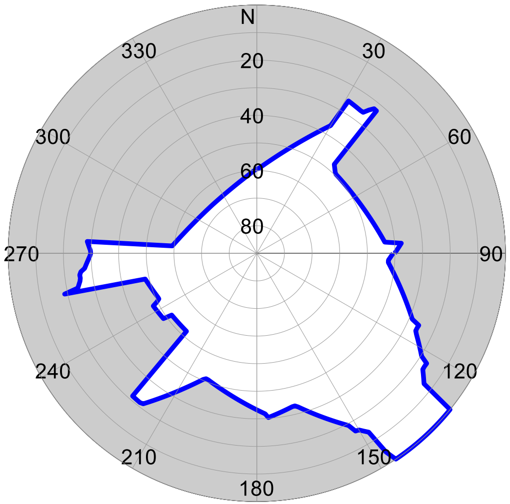


Extract surrounding 3D model



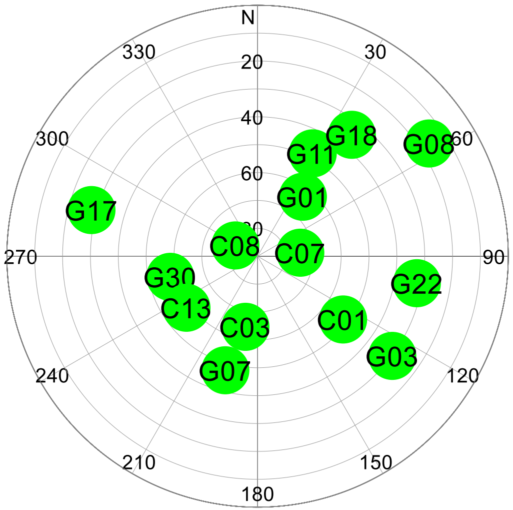
Identify blockage (Skymask)

Skymask – skyplot with building boundaries



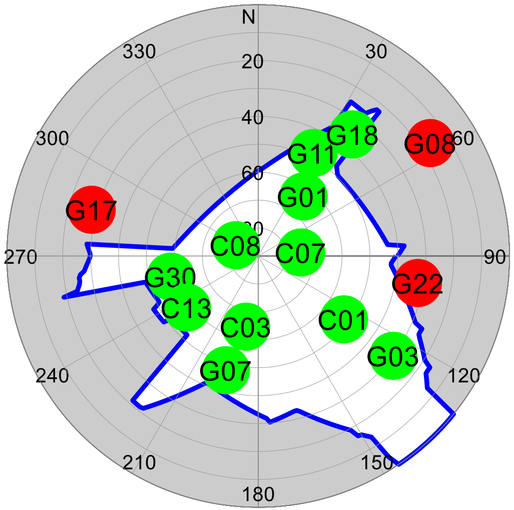
Skymask
by 3D building model

+



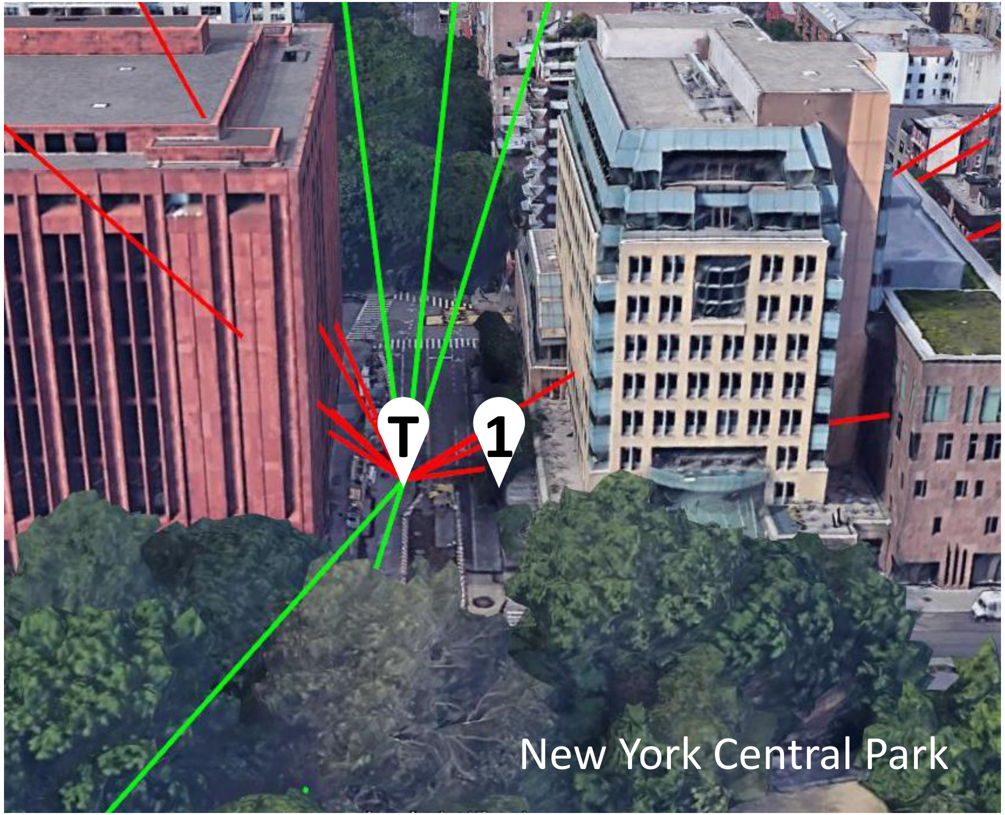
Skyplot
at receiver

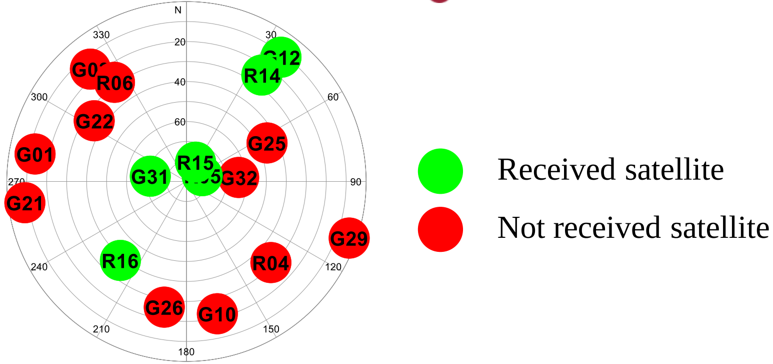
=



Visibility prediction

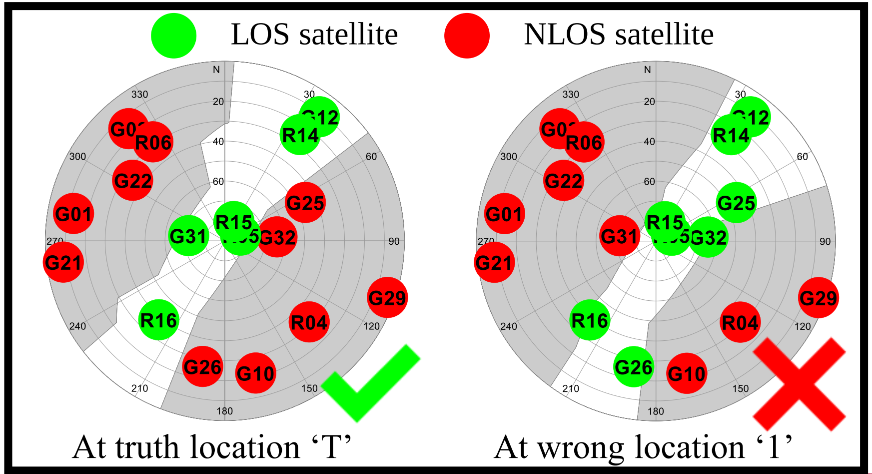
Shadow Matching





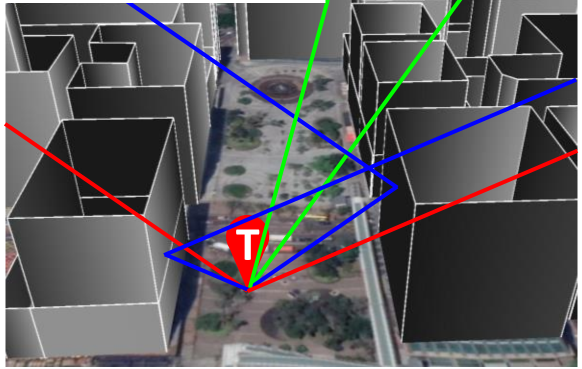
Received satellite

Satellite visibility prediction

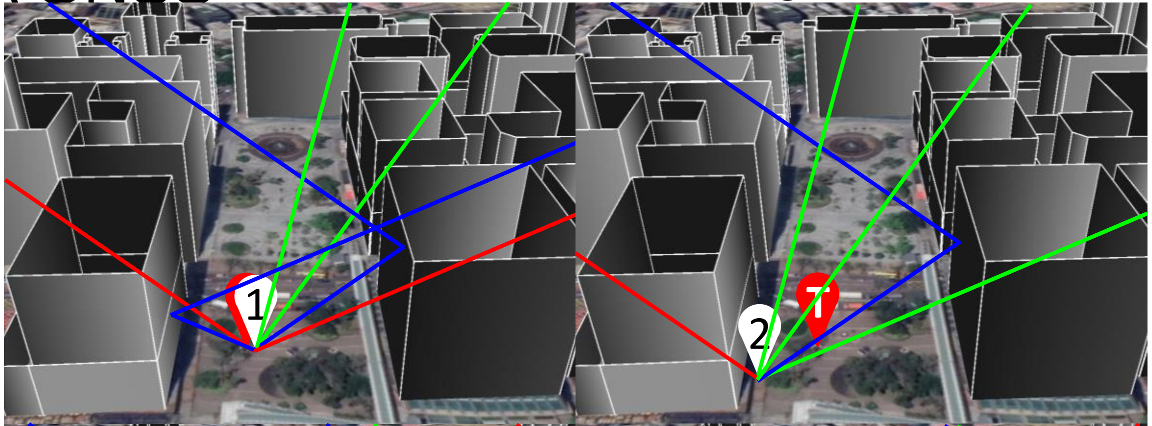


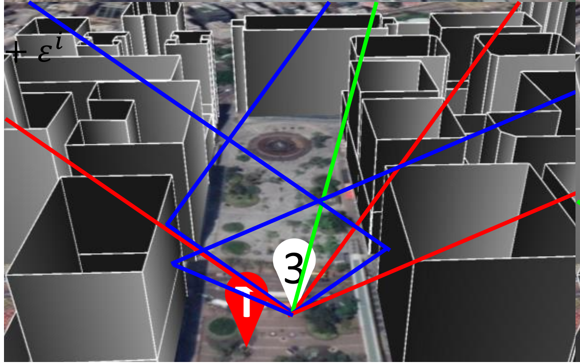
[1] Groves, P. (2011). Shadow Matching: A New GNSS Positioning Technique for Urban Canyons. *Journal of Navigation*, *64*(3), 417-430. doi:10.1017/S0373463311000087

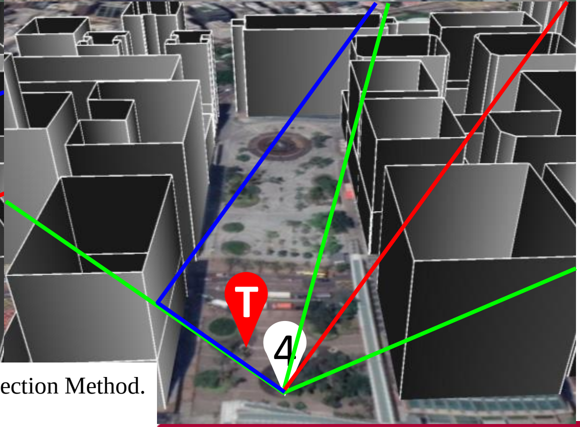
Ranging-Based 3DMA GNSS Measurements



Simulated Pseudorange







$$\rho_{j,refl}^{i} = D_j^i + c\left(\delta t_r - \delta t^i\right) + T^i + I^i + \varepsilon_{j,refl}^{i} + \varepsilon^i$$

Candidate	Similarity
1	Very high
2	High
3	Low
4	Low

[2] Ng, H.-F., Zhang, G., & Hsu, L.-T. (2020). A Computation Effective Range-based 3D Mapping Aided GNSS with NLOS Correction Method. *Journal of Navigation*, 1-21.
[3] Hsu, L.-T., Hu, Y., & Kamijo, S. (2016). 3D building model-based pedestrian positioning method using GPS/GLONASS/QZSS and its reliability calculation. *GPS Solutions*, 20(3), 413-428.

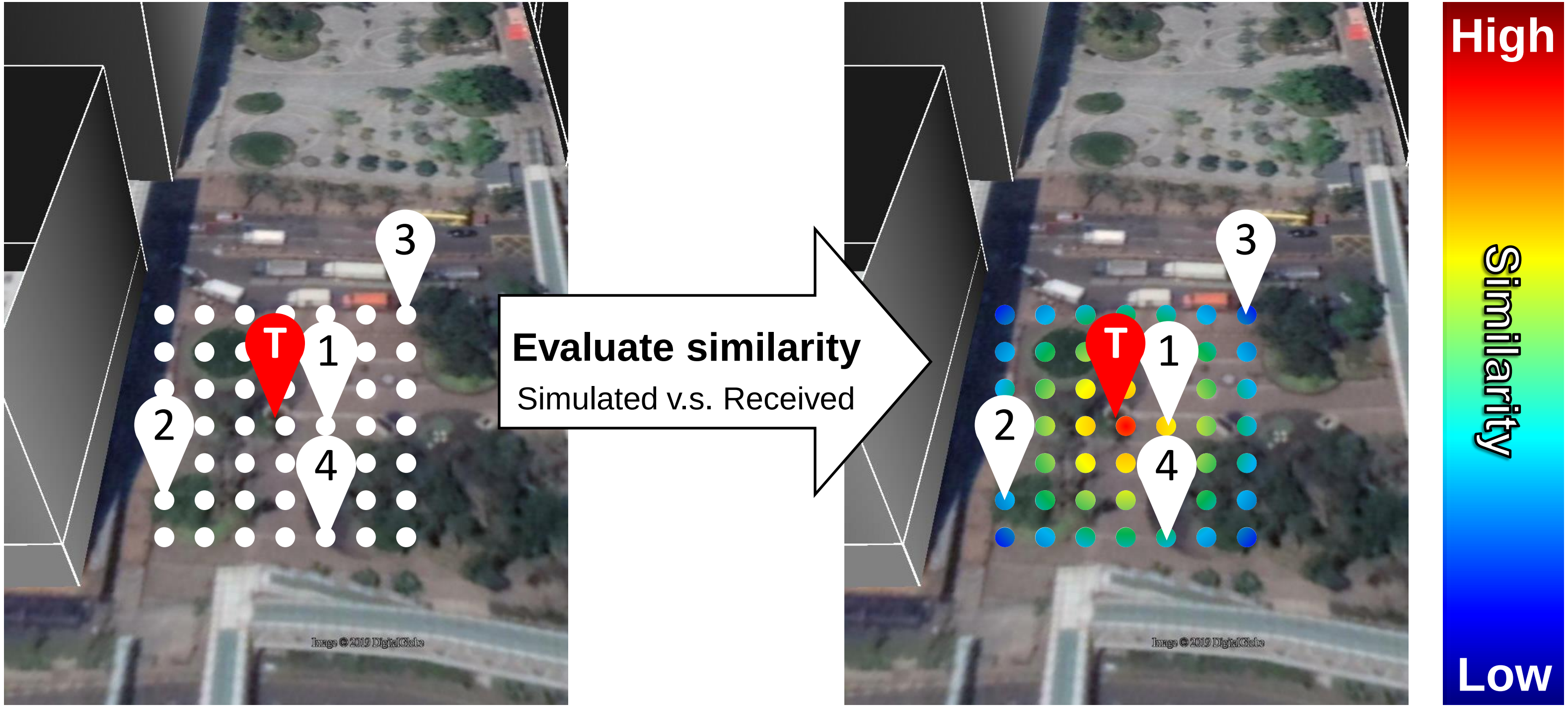
Shadow Matching v.s. Ranging Based 3DMA

Criteria	Shadow Matching [1]	Skymask 3DMA [2] & Ray-Tracing 3DMA [3,4]	Likelihood-Based Ranging GNSS [5]
Input parameters	Satellite position Positioning candidate position and its skymask	Satellite position Positioning candidate position and its enhanced skymask	Pseudorange difference
Similarity basis	Satellite visibility	Pseudorange	Pseudorange
Correction based	/	Geometry	Statistical
Available correction	/	Reflection delay	Reflection delay and some noise
Uncertainty	Satellite distribution, especially satellite near building edge	Building model accuracy	The healthiness of selected master satellite

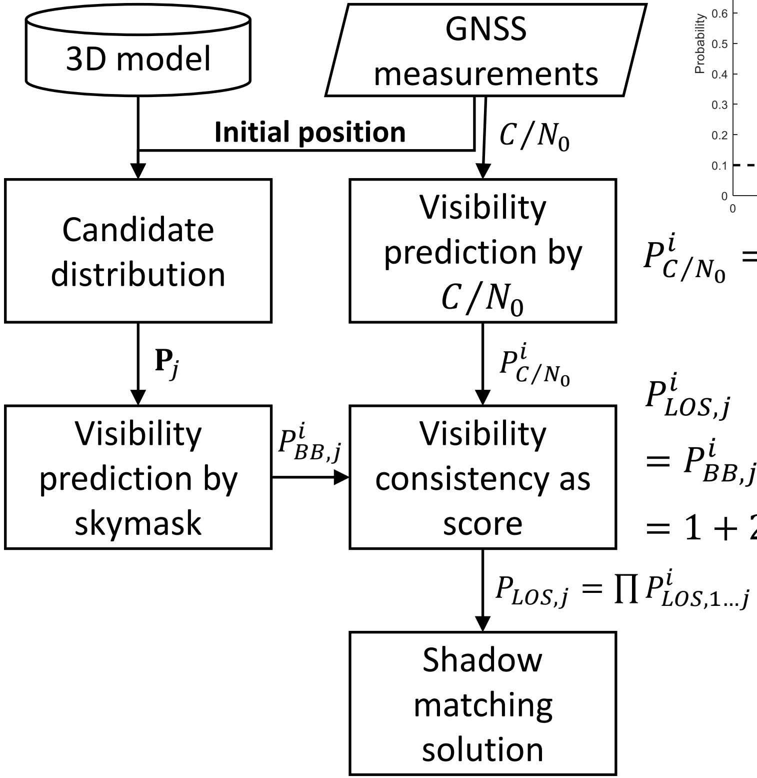
These can be combined and integrating with L5-band measurements!! [6]

[1] Groves, P. (2011). Shadow Matching: A New GNSS Positioning Technique for Urban Canyons. *Journal of Navigation, 64*(3), 417-430. doi:10.1017/S0373463311000087
[2] Ng, H.-F., Zhang, G., & Hsu, L.-T. (2020). A Computation Effective Range-based 3D Mapping Aided GNSS with NLOS Correction Method. *Journal of Navigation*, 1-21.
[3] Hsu, L.-T., Hu, Y., & Kamijo, S. (2016). 3D building model-based pedestrian positioning method using GPS/GLONASS/QZSS and its reliability calculation. *GPS Solutions*, 20(3), 413-428.
[4] Miura, S., Hsu, L.-T., & Chen, F. (2015). GPS Error Correction With Pseudorange Evaluation Using Three-Dimensional Maps. IEEE Transactions on Intelligent Transportation Systems, 16(6), 3104-3115.
[5] Groves, P. D., Zhong, Q., Faragher, R., & Esteves, P. (2020). Combining Inertially-aided Extended Coherent Integration (Supercorrelation) with 3D-Mapping-Aided GNSS. *ION GNSS+ 2020*.
[6] Ng, H.F., Zhang, G., L., Y. & Hsu, L. (2021). Urban Positioning: 3D Mapping Aided GNSS using Dual-Frequency Pseudorange Measurements from Smartphones. *Journal of Institute of Navigation*. (Accepted).

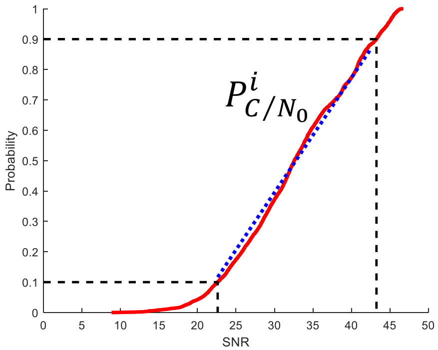
Positioning Hypothesis Candidate-Based Evaluation



Candidate-Based Shadow Matching

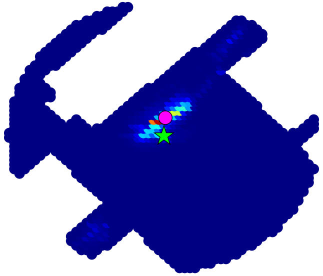


$$P^i_{BB,j} = \begin{cases} P_{max} & LOS \\ P_{min} & NLOS \end{cases}$$



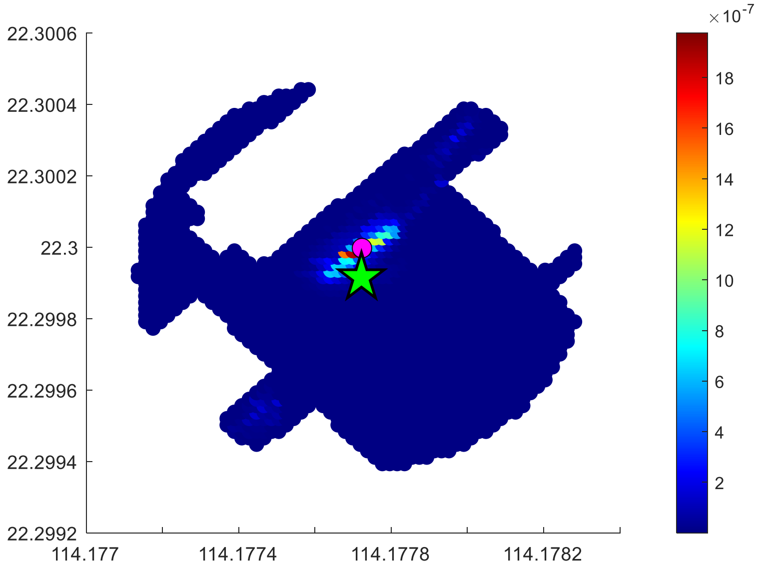
$$P^i_{C/N_0} = \mathrm{a}_0 + \mathrm{a}_1(C/N_0) + \mathrm{a}_2(C/N_0)^2$$

$$P^i_{LOS,j} = P^i_{BB,j} \times P^i_{C/N_0} + \left(1 - P^i_{BB,j}\right)\left(1 - P^i_{C/N_0}\right) = 1 + 2P^i_{BB,j}P^i_{C/N_0} - P^i_{BB,j} - P^i_{C/N_0}$$



[5] Groves, P. D., Zhong, Q., Faragher, R., & Esteves, P. (2020). Combining Inertially-aided Extended Coherent Integration (Supercorrelation) with 3D-Mapping-Aided GNSS. *ION GNSS+ 2020*.

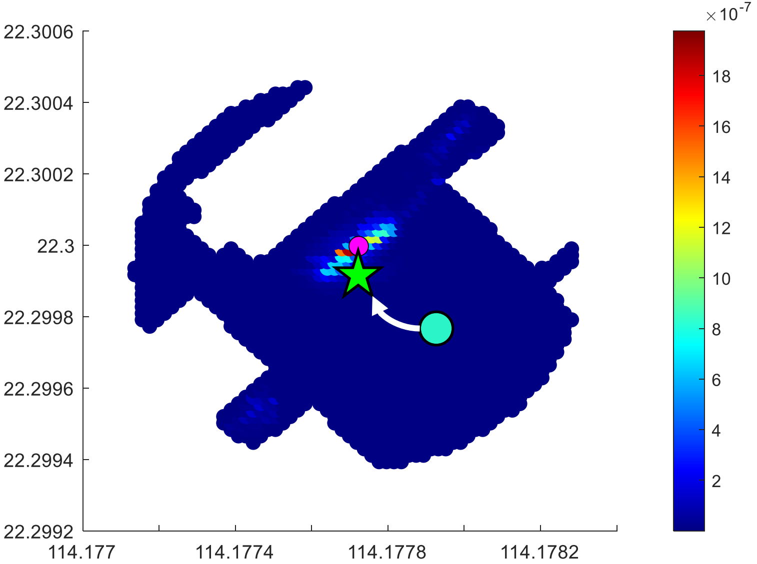
Candidate-Based Shadow Matching



Limitations…

1. Unwanted computation load created (blue area)
2. Candidates need to cover real location to achieve best performance

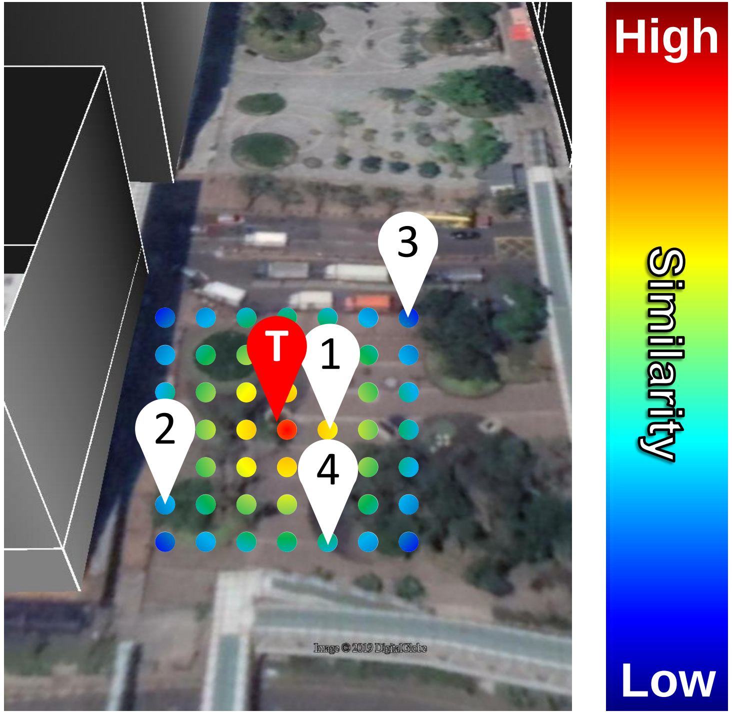
Candidate-Based Shadow Matching



What if…

we can identify <u>solution directly</u> by considering the problem as **optimization problem**?

Mathematical Expression



Objective:

Find a position that best match

measurements and prediction

$$\mathbf{x} = \underset{\mathbf{x}}{\operatorname{argmin}} \left\| \textit{received measurements} - \textit{estimated measurements}_{\mathbf{x}} \right\|$$

$$\mathbf{x} = \underset{\mathbf{x}}{\operatorname{argmin}} \| y - \hat{y}_{\mathbf{x}} \| = \underset{\mathbf{x}}{\operatorname{argmin}} \| y - F(\mathbf{x}) \|$$

A Step Back to Shadow Matching

Measurements dependent

Objective function, visibility consistency: $P^i_{LOS,j} = P^i_{BB,j} \times P^i_{C/N_0} + (1 - P^i_{BB,j})(1 - P^i_{C/N_0})$

Geometry dependent

Visibility prediction by skymask: $P^i_{BB,j} = \begin{cases} P_{max} & LOS \\ P_{min} & NLOS \end{cases}$

$$P^i_{BB}(\mathbf{x}) \propto [EL^i - BB(AZ^i)]$$

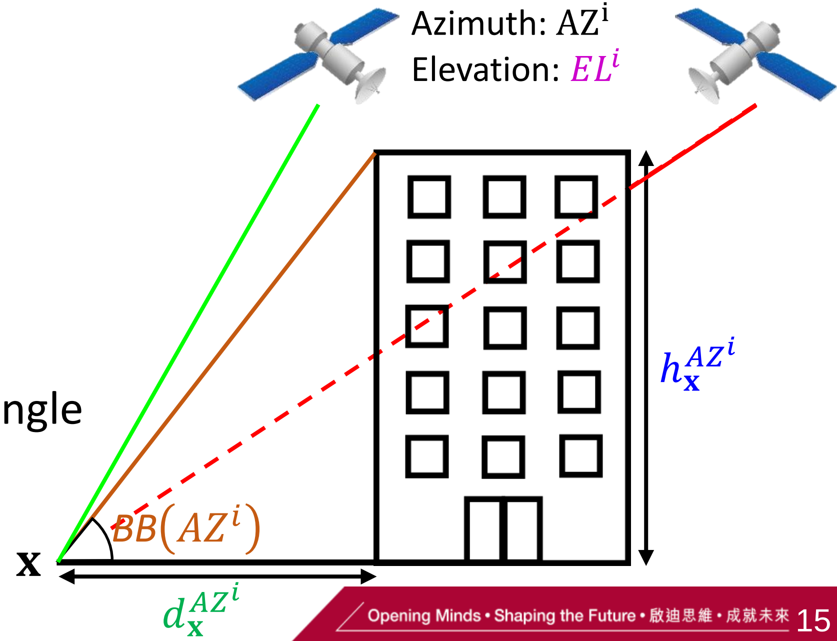
$$EL^i - BB(AZ^i) = EL^i - \tan^{-1}\frac{h_{\mathbf{x}}^{AZ^i}}{d_{\mathbf{x}}^{AZ^i}} = \begin{cases} LOS & > 0 \\ NLOS & \leq 0 \end{cases}$$

$$P^i_{BB}(\mathbf{x}) \propto \tanh\left(EL^i - BB(AZ^i)\right) \times \tau + \frac{1}{2}$$

Scaling factor

Move position **x** to change the building elevation angle

	$EL^i - \mathrm{BB}(AZ^i)$	$\mathrm{BB}(AZ^i)$	$d_{\mathbf{x}}^{AZ,i}$
LOS → NLOS	negative	↑	↓
NLOS → LOS	positive	↓	↑



Modelling 3D Model Prediction with State $\mathbf{x}$

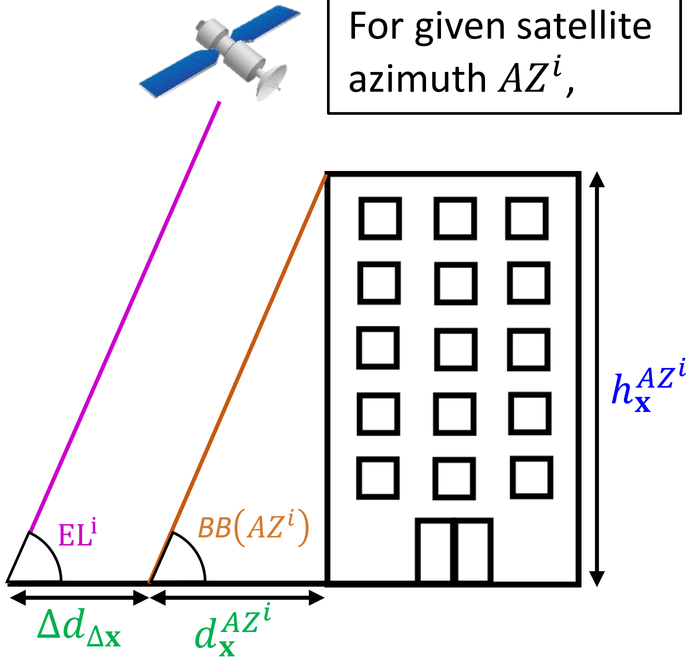
State (position) $\mathbf{x} = (East, North)$

$$P_{BB}^{i}(\mathbf{x}) \propto \tanh\left(\mathrm{EL}^{\mathrm{i}} - BB\left(\mathrm{AZ}^{\mathrm{i}}\right)\right)$$

$$P_{BB}^{i}(\mathbf{x}) \propto \tanh\left(\mathrm{EL}^{\mathrm{i}} - \tan^{-1}\frac{h_{\mathbf{x}}^{\mathrm{AZ^i}}}{\left|d_{\mathbf{x}}^{\mathrm{AZ^i}} + \Delta d_{\Delta\mathbf{x}}\right|}\right)$$

$$P_{BB}^{i}(\mathbf{x}) = \tanh\left[\alpha\left(\mathrm{EL}^{\mathrm{i}} - \tan^{-1}\frac{h_{\mathbf{x}}^{\mathrm{AZ^i}}}{\left|d_{\mathbf{x}}^{\mathrm{AZ^i}} + \Delta d_{\Delta\mathbf{x}}\right|}\right)\right] \times \tau + \frac{1}{2}$$

For given satellite azimuth AZ^i,



Modelling 3D Model Prediction with State x

$$P_{BB}^{i}(\mathbf{x}) = \tanh\left[\alpha\left(\mathrm{EL}^{\mathrm{i}} - \tan^{-1}\frac{h_{\mathbf{x}}^{\mathrm{AZ^i}}}{\left|d_{\mathbf{x}}^{\mathrm{AZ^i}}+\Delta d_{\Delta\mathbf{x}}\right|}\right)\right]\times\tau+\frac{1}{2}$$

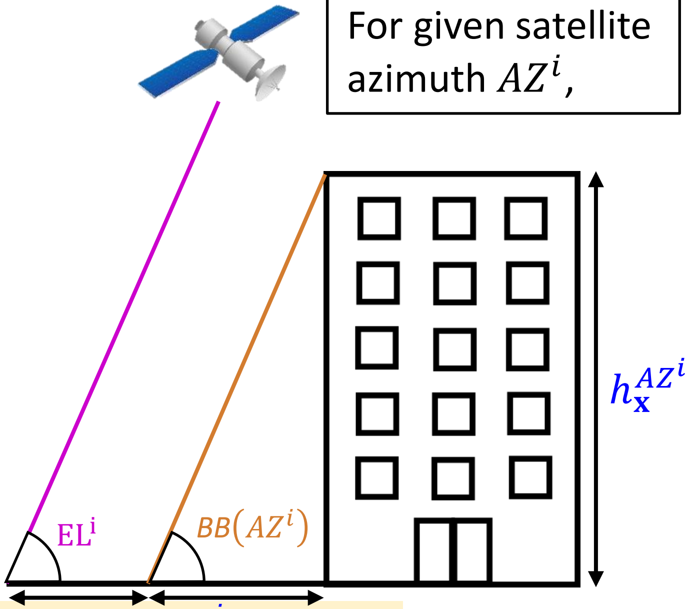
Disassemble $\left|d_{\mathbf{x}}^{\mathrm{AZ^i}}+\Delta d_{\Delta\mathbf{x}}\right|$ into <u>2D-case</u>,

$$\left|d_{\mathbf{x}}^{\mathrm{AZ,i}}+\Delta d_{\Delta\mathbf{x}}\right| = \sqrt{E_{total}^{2}+N_{total}^{2}}$$

$$\Delta E_{total} = -d_{\mathbf{x}}^{AZ^i}\sin\left(AZ^i\right)+\Delta E$$

$$\Delta N_{total} = -d_{\mathbf{x}}^{AZ^i}\cos\left(AZ^i\right)+\Delta N$$

For given satellite azimuth AZ^i,

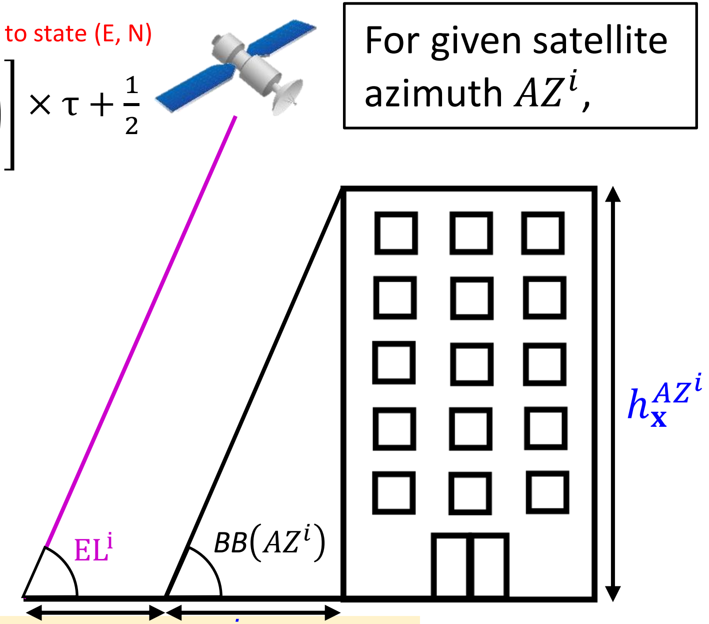


East:	ΔE	$d_{\mathbf{x}}^{AZ^i}\sin\left(AZ^i\right)$
North:	ΔN	$d_{\mathbf{x}}^{AZ^i}\cos\left(AZ^i\right)$

Modelling 3D Model Prediction with State $\mathbf{x}$

Blue: Retrieving from database based on the state (E, N) and satellite azimuth AZ^{i} and elevation EL^{i} angle
Red: Function / variable related to state (E, N)

$$P_{BB}^{i}(\mathbf{x}) = \tanh\left[\alpha\left(\mathrm{EL}^{\mathrm{i}} - \tan^{-1}\frac{h_{\mathbf{x}}^{\mathrm{AZ}^{\mathrm{i}}}}{\sqrt{\left(\Delta E - d_{\mathbf{x}}^{AZ,i}\sin(\mathrm{AZ}^{\mathrm{i}})\right)^2 + \left(\Delta N - d_{\mathbf{x}}^{AZ,i}\cos(\mathrm{AZ}^{\mathrm{i}})\right)^2}}\right)\right] \times \tau + \frac{1}{2}$$



Disassemble $\left|d_{\mathbf{x}}^{\mathrm{AZ}^{\mathrm{i}}} + \Delta d_{\Delta\mathbf{x}}\right|$ into **2D-case**,

$$\left|d_{\mathbf{x}}^{\mathrm{AZ,i}} + \Delta d_{\Delta\mathbf{x}}\right| = \sqrt{E_{total}^2 + N_{total}^2}$$

$$\Delta E_{total} = -d_{\mathbf{x}}^{AZ^i}\sin\left(AZ^i\right) + \Delta E$$

$$\Delta N_{total} = -d_{\mathbf{x}}^{AZ^i}\cos\left(AZ^i\right) + \Delta N$$

East:	ΔE	$d_{\mathbf{x}}^{AZ^i}\sin\left(AZ^i\right)$
North:	ΔN	$d_{\mathbf{x}}^{AZ^i}\cos\left(AZ^i\right)$

Summarize on Objective Function & Nonlinear Least Squares

Optimization problem $\mathbf{x} = \underset{\mathbf{x}}{\operatorname{argmin}} \| y - F(\mathbf{x}) \|$

State (position) $\mathbf{x} = [x_1, x_2]^T = [\Delta E, \Delta N\,]^T$

Blue: Retrieving from database based on the state (E, N) and satellite azimuth $\mathrm{AZ^i}$ and elevation $\mathrm{EL^i}$ angle

Red: Function / variable related to state (E, N)

Measurements $P^i_{C/N_0} = \mathrm{a}_0 + \mathrm{a}_1(C/N_0) + \mathrm{a}_2(C/N_0)^2$

Estimations $P^i_{BB}(\mathbf{x}) = \tanh\left[\alpha\left(\mathrm{EL^i} - \tan^{-1}\frac{h_{\mathbf{x}}^{\mathrm{AZ,i}}}{\sqrt{\left(\Delta E - d_{\mathbf{x}}^{AZ,i}\sin(\mathrm{AZ^i})\right)^2 + \left(\Delta N - d_{\mathbf{x}}^{AZ,i}\cos(\mathrm{AZ^i})\right)^2}}\right)\right] \times \tau + \frac{1}{2}$

Visibility consistency $P^i_{LOS} = P^i_{BB} \times P^i_{C/N_0} + \left(1 - P^i_{BB}\right)\left(1 - P^i_{C/N_0}\right)$

Objective function $f^i(\mathbf{x}) = -\log\left(P^i_{LOS}\right)$

Objective function $\mathbf{F}(\mathbf{x}) = \left[f^1(\mathbf{x}) \ldots f^i(\mathbf{x})\right]^T$

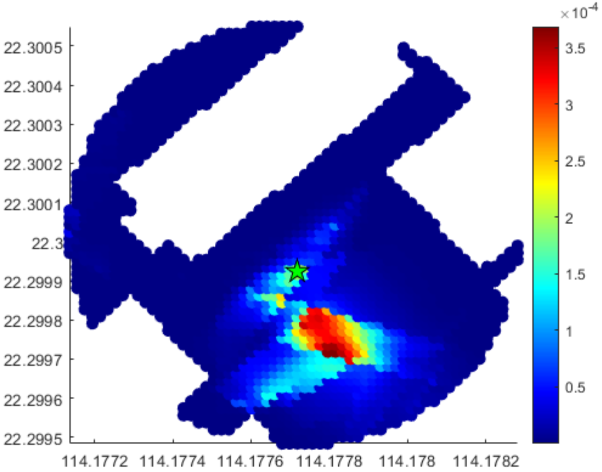
Error function $e(\mathbf{x}) = \frac{1}{2}\|\mathbf{F}(\mathbf{x})\|^2 = \frac{1}{2}\mathbf{F^T F}$

Jacobian matrix $\mathbf{J}(\mathbf{x}) = \nabla\mathbf{F}(\mathbf{x}) = \begin{bmatrix} \frac{\partial f^1}{\partial E} & \frac{\partial f^1}{\partial N} \\ \vdots & \vdots \\ \frac{\partial f^i}{\partial E} & \frac{\partial f^i}{\partial N} \end{bmatrix}$

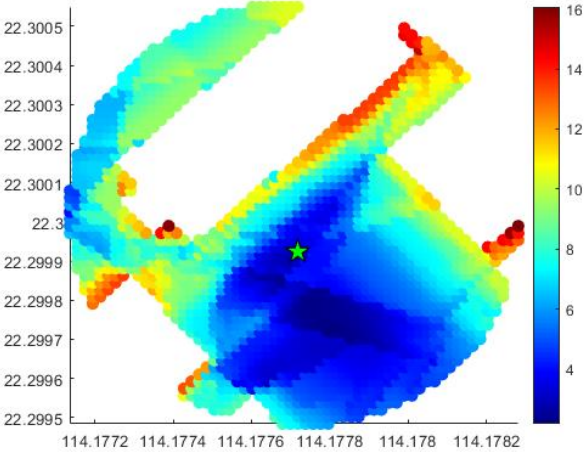
Gauss-Newton method $\mathbf{x^n} = \mathbf{x^{n-1}} - \left[\mathbf{J}\left(\mathbf{x^{n-1}}\right)^T \mathbf{J}\left(\mathbf{x^{n-1}}\right)\right]^{-1} \cdot \mathbf{J}\left(\mathbf{x^{n-1}}\right) \cdot \mathbf{F}\left(\mathbf{x^{n-1}}\right)$

[7] Agarwal, S., & Mierle, K. (2012). Ceres solver: Tutorial & reference. *Google Inc*, *2*(72), 8.

Change of estimated measurements

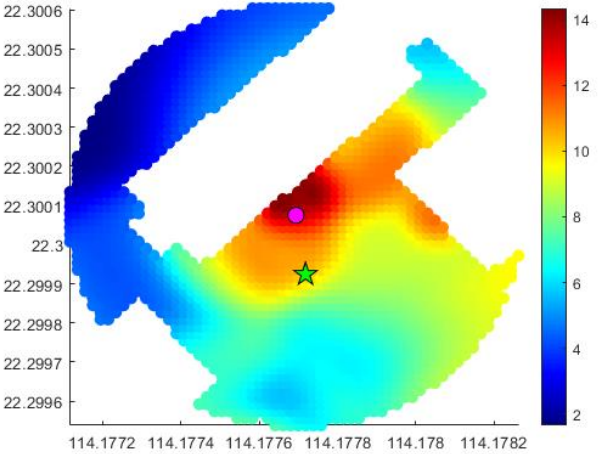


Candidate-based $P_{LOS,j}$



Proposed modelling $e(\mathbf{x}) = \frac{1}{2}\|\mathbf{F}(\mathbf{x})\|^2$

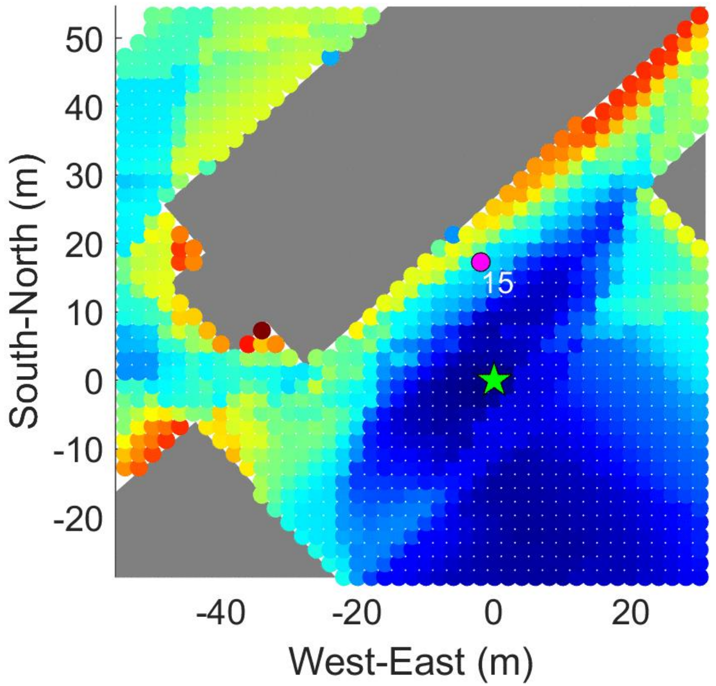
$h_{\mathbf{x}}^{\mathrm{AZ,i}}$ & $d_{\mathbf{x}}^{\mathrm{AZ,i}}$ based on <u>each location</u>, $\mathbf{x}$

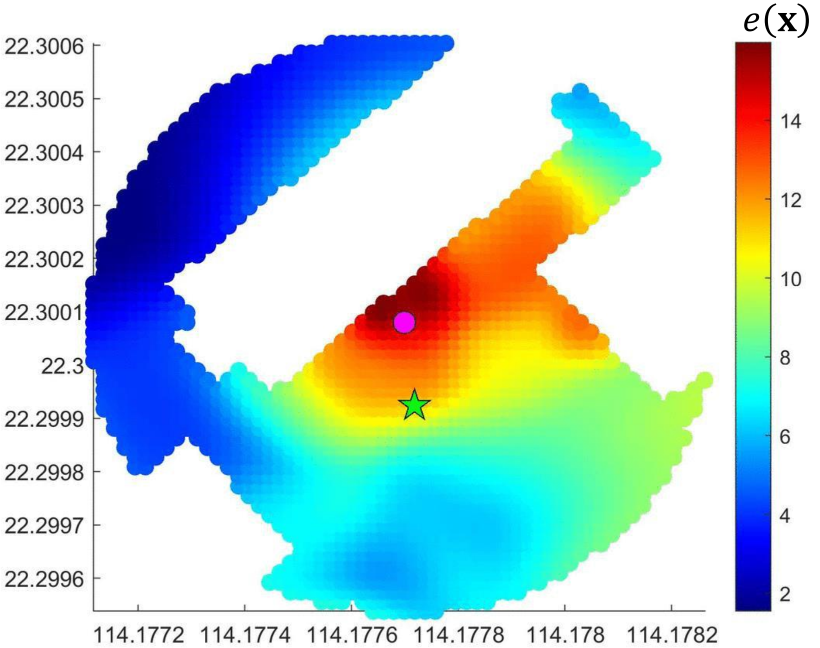


Proposed modelling $e(\mathbf{x}) = \frac{1}{2}\|\mathbf{F}(\mathbf{x})\|^2$

$h_{\mathbf{x}}^{\mathrm{AZ,i}}$ & $d_{\mathbf{x}}^{\mathrm{AZ,i}}$ based on <u>initial location</u>, $\mathbf{x}^{\mathbf{0}}$

Is location dependent variables important?

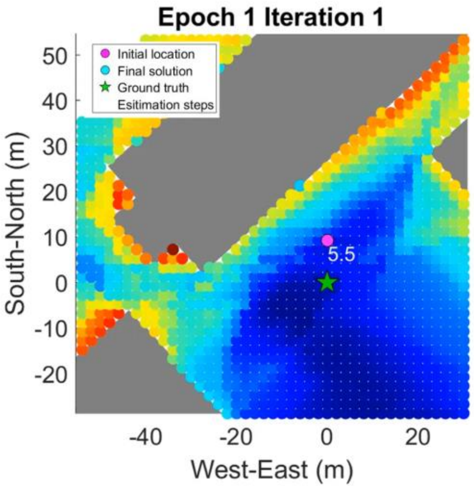




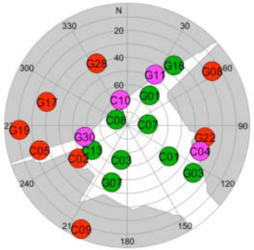
Proposed modelling $e(\mathbf{x}) = \frac{1}{2}\|\mathbf{F}(\mathbf{x})\|^2$

$h_{\mathbf{x}}^{\mathrm{AZ,i}}$ & $d_{\mathbf{x}}^{\mathrm{AZ,i}}$ based on initial location, $\mathbf{x}^{\mathbf{0}}$

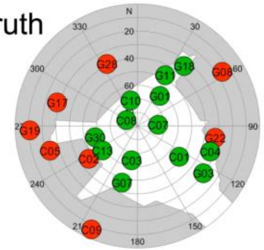
Experiment Results (1)



Estimation

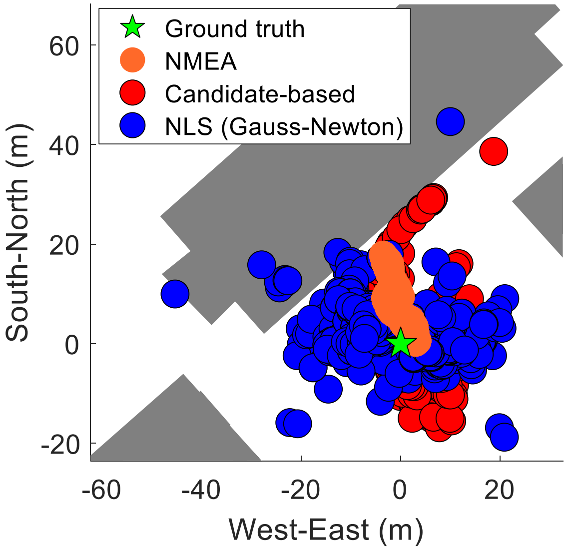


Ground Truth



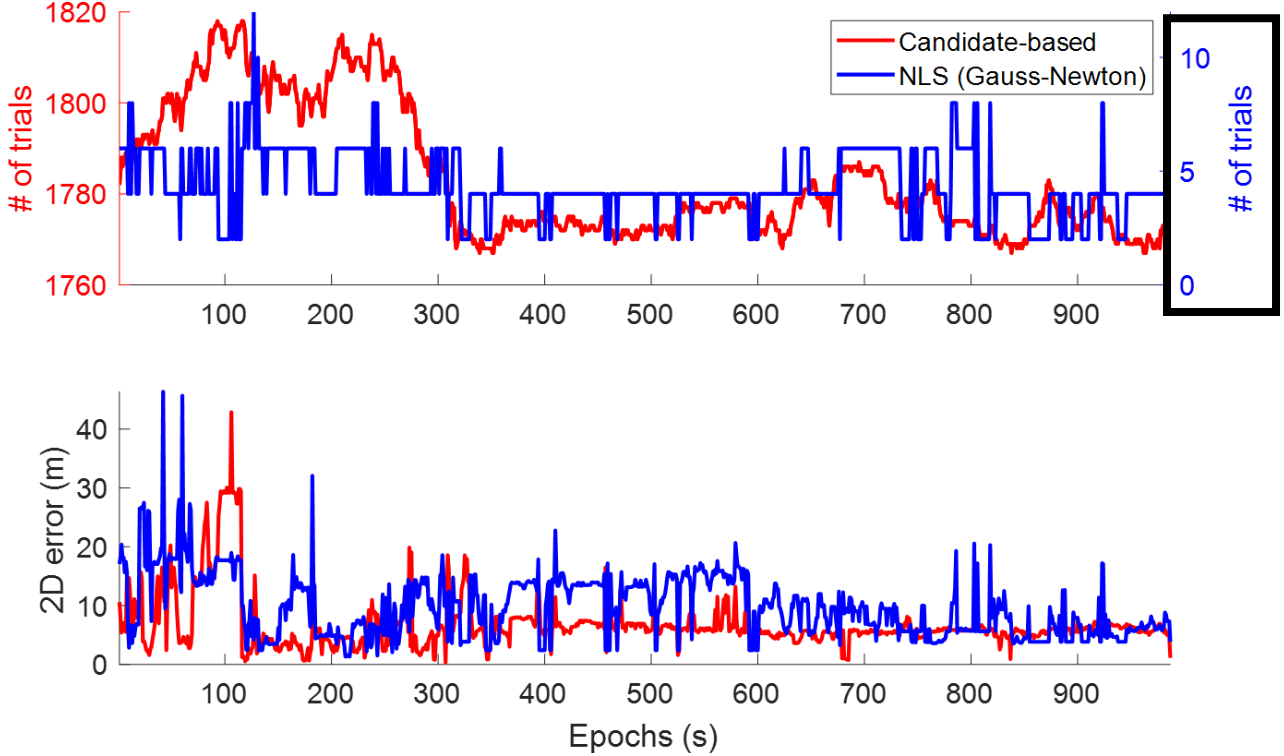
GPS(L1)+BDS(B1)
Candidate distribution
Radius: 40m
Separation: 2m
Initial location: NMEA

(Mi 8)

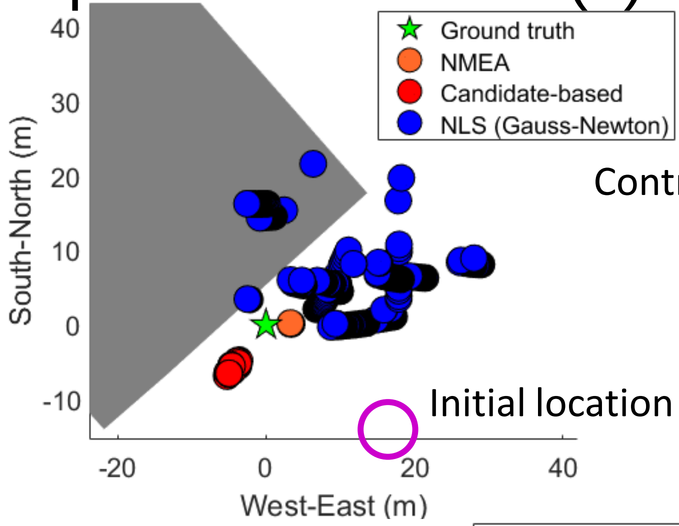


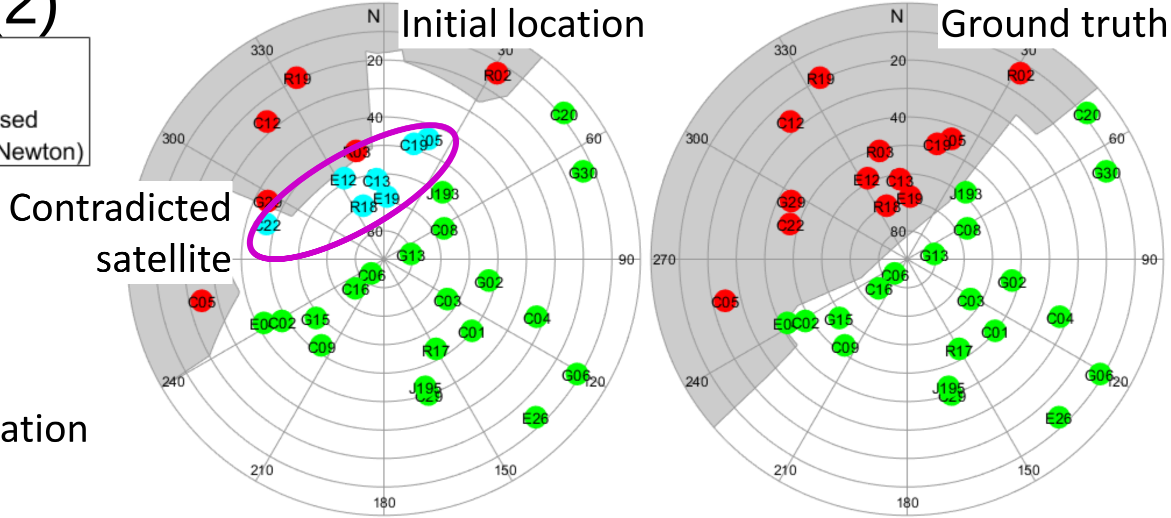
Unit: meter	NEMA	Candidate-based	Gauss-Newton Algorithm
RMS	7.57	8.26	11.11
Mean	6.26	6.78	9.74
STD	4.25	4.73	5.33
MAX	18.22	42.90	46.39
MIN	2.90	0.31	1.32

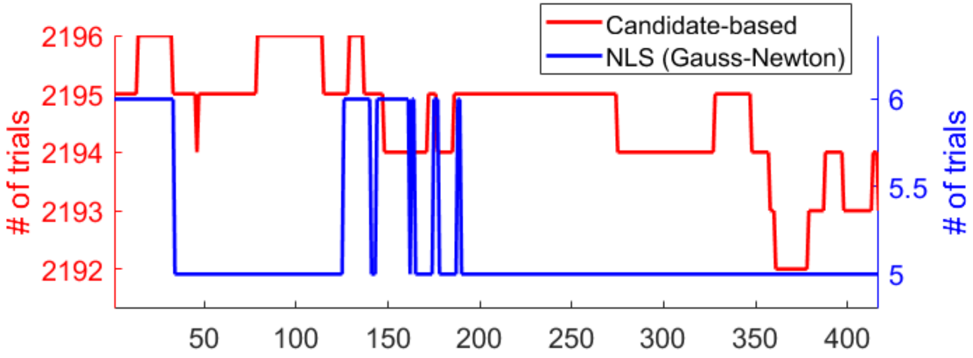
Experiment Results (1)



Experiment Results (2)

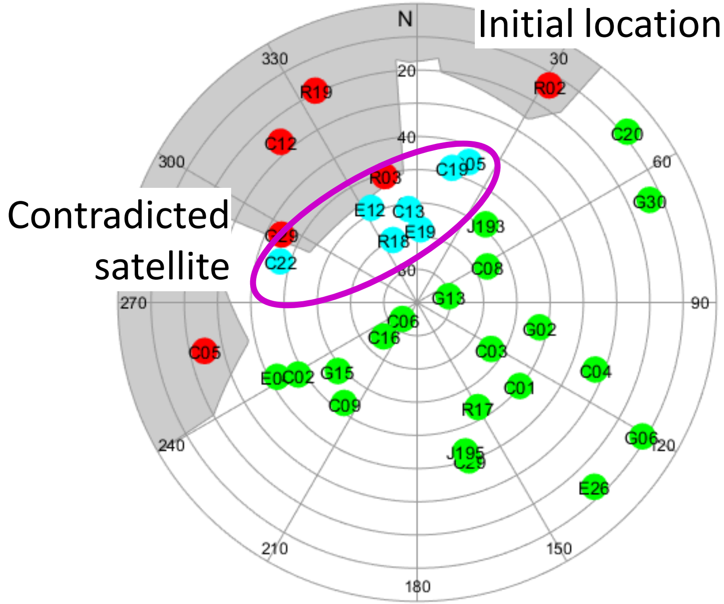


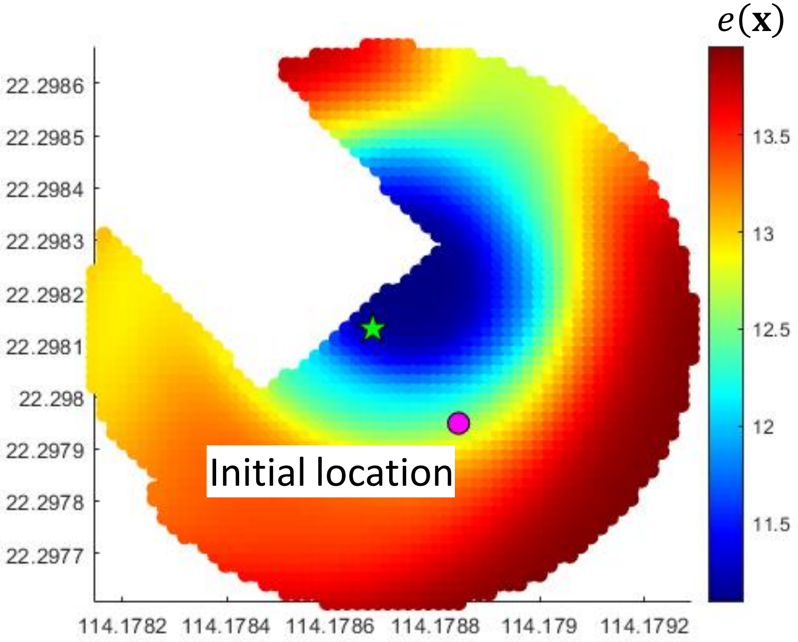




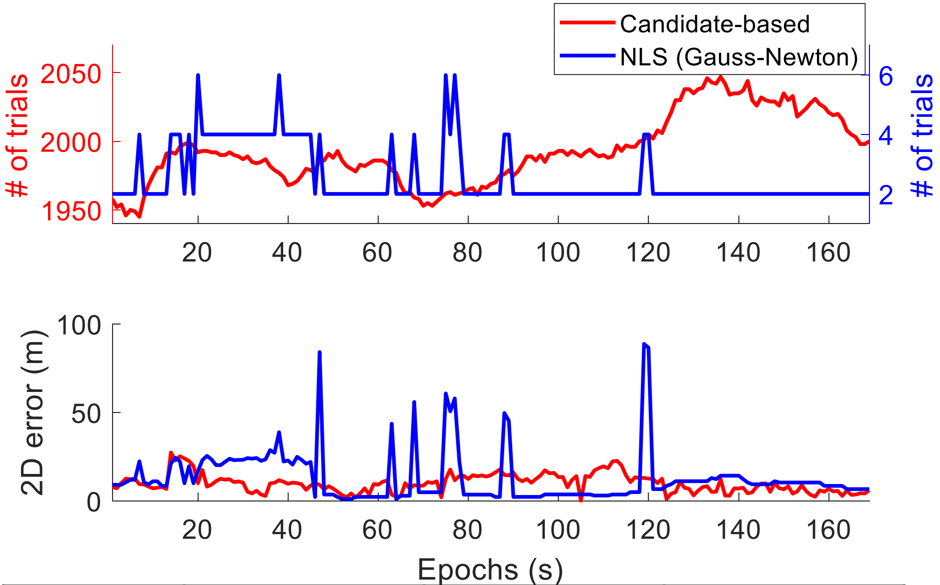
Unit: meter	NEMA	Candidate-based	Gauss-Newton Algorithm
RMS	3.34	7.87	16.77
Mean	3.34	7.83	15.42
STD	0.07	0.80	6.59
MAX	3.46	8.68	30.12
MIN	3.26	5.96	4.05

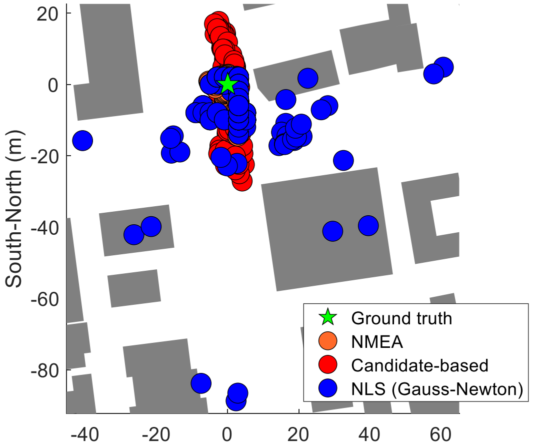
Relationship between satellite and building geometry



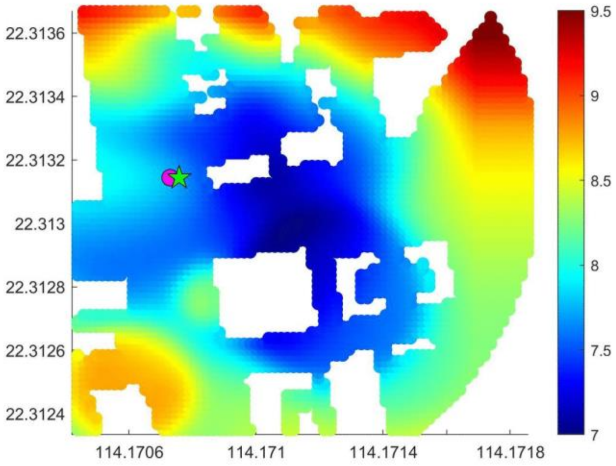


Experiment Results (3)

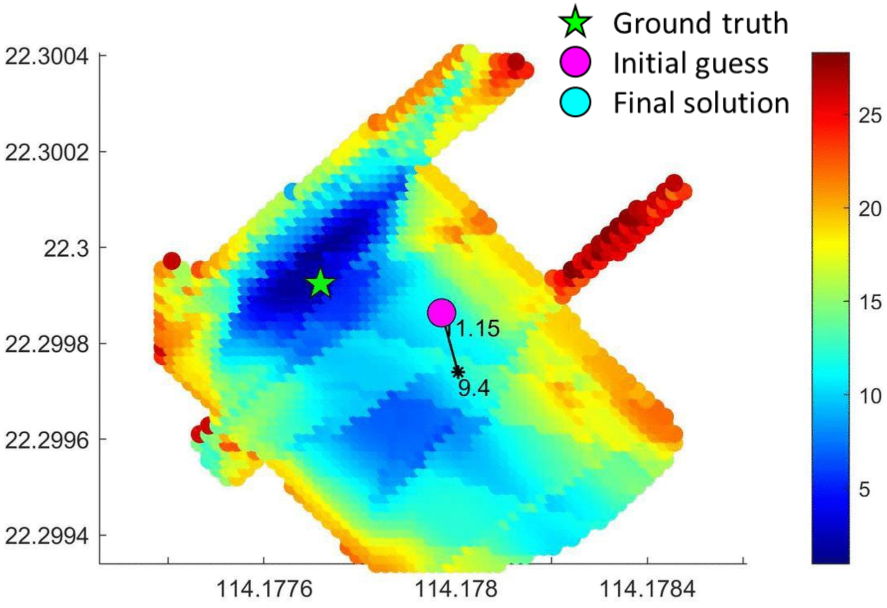




Unit: meter	NEMA	Candidate-based	Gauss-Newton Algorithm
RMS	7.34	11.19	19.89
Mean	6.42	9.91	13.07
STD	3.57	5.21	15.04
MAX	14.73	27.40	88.75
MIN	1.19	0.15	0.94



Limitation: Local Minimum Issue (same for particle-based Shadow matching)

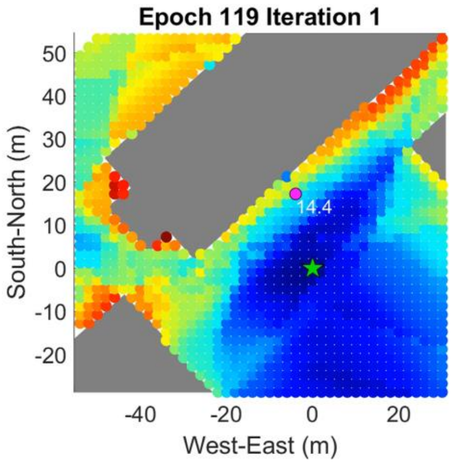


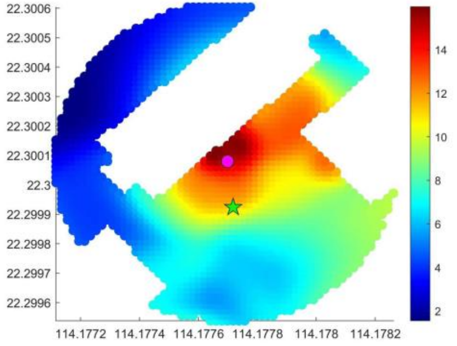
When initial point is far and local minima occurred, solution may not be converged.

Conclusions

- Modelling the objective function of shadow matching with position state $\mathbf{x}$
- Optimizing the shadow matching with Gauss-Newton method
- Reduce the number of trials (computational load)
- Actual smartphone data obtains positioning error within 20m

- Location dependent variables ($h_{\mathbf{x}}^{\mathrm{AZ,i}}$ & $d_{\mathbf{x}}^{\mathrm{AZ,i}}$) is important for proposed modelling
- Satellite distribution will affect the performance





Future Work

Solution wise:

- Improving the modelling, especially environment dependent variables
- Modelling other 3DMA GNSS, such as ray-tracing and skymask 3DMA

Implementation wise:

- Integrating with different open-source library.
  E.g. Ceres Solver, GTSAM, GraphGNSSLib

Epoch wise:

- Correlating the snapshot-based solution with time to become FGO.
  E.g. integrating Doppler measurements

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Thank you for your attention

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